Code No. : 6121

FACULTY OF ENGINEERING
B.E. 2/4 (ECE/M/P/CSE) II Semester (Supple.) Examination, December 2009
MATHEMATICS – IV

Time: 3 Hours] [Max. Marks : 75

Note : Answer **all** questions of Part **A**. Answer **five** questions from Part **B**.

PART – A **25 Marks**

1. Define an analytic function. Is $f(z) = e^z$ analytic ? **2**

2. Choose the correct answer. $\int_0^{1+i} (x^2 - iy)dz$ along the path $y = x$ is

   a) $\frac{5}{6} + \frac{1}{6}i$ b) $\frac{5}{6} - \frac{1}{6}i$

   c) $\frac{5}{6}i - \frac{1}{6}$ d) None. **2**

3. Find the image of the circle $|z| = 2$ under the transformation $w = z + 3 + 2i$. **3**

4. Expand function sin z as a Taylor's series about $z = \frac{\pi}{4}$. **3**

5. Define correlation between two random variables X and Y. **2**

6. If X is a random variable and if 'a' and 'b' are constants then show that $V(aX + b) = a^2V(X)$. **3**

7. Write down basic properties of the normal distribution. **2**

8. Explain the term type I error, type II error and the level of significance. **3**

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9. The mean and variance of Gamma distribution. **2**

a) Are same b) Cannot be same

c) Are some times equal d) Are equal in the limiting case, as $n \to \infty$.

10. Find the students 't' for the following variable values in a sample of $10, -6, -4, -1, -1, 0, 1, 1, 3, 4, 5$ taking the mean of universe to be zero. **3**

PART – B **50 Marks**

11. a) Derive Cauchy - Riemann equations in polar coordinates form. **5**

b) Show that the function $u = e^{-2xy} \sin(x^2 - y^2)$ is harmonic. Conjugate function v and express $u + iv$ as an analytic function of z. **5**

12. a) Show that the transformation $w = \frac{2z+3}{z-4}$ maps the circle $x^2 + y^2 - 4x = 0$ onto the straight line $4\mu + 3 = 0$.

b) State and prove Cauchy's integral formula. **5**

13. State and prove residue theorem. Evaluate using residual theorem $\oint_C \frac{e^z}{(z+1)^2} dz$, where C is the circle $|z-1| = 3$. **10**

14. a) State Bayes theorem.

A bag X contains 2 white and 3 red balls and a bag Y contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag Y. **5**

b) Obtain the mean and variance of the random variable Y given that $Y = 3X + 4$ and that X is a random variable with its mean 6 and variance 4. **5**

15. a) Derive the mean, variance and m. g. f. of Poisson distribution. **5**

b) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. **5**

16. a) Establish the formula $\sigma_{x-y}^{2} = \sigma_x^2 + \sigma_y^2 - 2r\sigma_x\sigma_y$ where r is coefficient of correlation. **5**

b) The life time of electric bulbs for a random sample of 10 from a large consignment gave the following data :

Item	1	2	3	4	5	6	7	8	9	10
Life in '000 hrs.	4.2	4.6	3.9	4.1	5.2	3.8	3.9	4.3	4.4	5.6

Can we accept the hypothesis that the average life time of bulbs is 4000 hrs ? **5**

17. a) If X is a random variable which follows normal distribution with mean 12 and S.D 4, find the probability that

i) $X \geq 20$ ii) $0 \leq X \leq 12$

(Given that $\int_0^2 \phi(z)\,dz = 0.4772$ and $\int_0^3 \phi(z)\,dz = 0.4987$ **5**

b) Evaluate :

$\int_C z^2 dz$, , where C is the arc of the circle $|z| = 2$ from $\theta = 0$ to $\theta = \frac{\pi}{3}$. **5**
