

FACULTY OF ENGINEERING AND INFORMATICS
 B.E. I Year (Common to All Branches) (Old) Examination, January 2012
 MATHEMATICS – II

Time: 3 Hours]

[Max. Marks: 75

Note : Answer *all* questions from Part A. Answer *any five* questions from Part B.

PART – A

(25 Marks)

1. Integrating factor of $x \frac{dy}{dx} + y = x^3 y^6$ is _____ 2
2. Find the orthogonal trajectories of $x^2 + y^2 = a^2$. 3
3. Find the particular integral of $(D^2 + 36)x = 4 \cos 6t$. 2
4. Solve $(D^2 - 2D + 5)^2 y = 0$. 3
5. Find the Laplace transform of $\cos^2 4t$. 2
6. Find the inverse Laplace transform of $\frac{s^2}{(s^2 + 4)^2}$. 3
7. Find the polynomial $2x^2 + x + 3$ in terms of Legendre polynomials $P_n^{(x)}$. 3
8. The value of $J_{\frac{1}{2}}(x) =$ _____ 2
9. Evaluate $\int_0^{\infty} \sqrt{x} e^{-x^2} \cdot dx$ in terms of Gamma functions. 3
10. Show that $T_n(-x) = (-1)^n T_n(x)$. 2

PART – B

(5×10=50 Marks)

11. a) Solve $x \frac{dy}{dx} + y = x^2 y^2 \log x$. 5
- b) Find the equation of the family of all orthogonal trajectories of the family of circles which pass through the points $(2, 0), (-2, 0)$. 5



12. a) Solve $y'' + 4y' + 4y = 3\sin x + 4\cos x$, $y(0) = 1$ and $y'(0) = 0$. 5
- b) Solve by the method of variation of parameters $y'' + 6y' + 9y = \frac{e^{-3x}}{x}$. 5
13. a) Evaluate $L\left\{\int_0^t \frac{e^{-t} \sin t}{t} dt\right\}$. 5
- b) Using Convolution theorem, evaluate $L^{-1}\left\{\frac{s}{(s^2 + 1)(s^2 + 4)}\right\}$. 5
14. a) Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$. 5
- b) Show that $\int_{-1}^1 p_m(x) P_n(x) dx = 0$, $m \neq n$. 5
15. a) Prove that $\beta(m, n) = \frac{\Gamma(m) \Gamma(n)}{\Gamma(m+n)}$. 5
- b) Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. 5
16. a) Prove that $T_{n+1}(x) - 2x T_n(x) + T_{n-1}(x) = 0$. 5
- b) State and prove the generating function of Chebyshev polynomials of $U_n(x)$. 5
17. Find the power series solution about the point $x_0 = 2$ of the equation $y'' + (x-1)y' + y = 0$. 10