1/4 Tefin

Code No. : **5010/O**

FACULTY OF ENGINEERING & INFORMATICS

B.E. I Year (Common to all Branches) Examination, January 2012

MATHEMATICS – I (Old)

Time: 3 Hours] [Max. Marks: 75

***Note**: Answer **all** questions of Part **A**. Answer **five** questions from Part **B**.*

PART – A **(25 Marks)**

1. Determine the rank of $\begin{bmatrix} 1 & 2 & 3 & 2 \\ -1 & 1 & 3 & -5 \\ 2 & 3 & 4 & 5 \end{bmatrix}$. **3**
2. Test whether the vectors (1, 1, 2), (1, 0, 1), (2, 3, 5) are dependent or not. **2**
3. State p-series test. **2**
4. Discuss the convergence of the series $\sum \frac{1}{n(n+1)}$. **3**
5. Verify Lagrange's mean value theorem for $f(x) = x^2$ in [1, 5]. **3**
6. Find the envelope of the family of straight lines $y = ax + a^2$, a is a parameter. **2**
7. Find the vertical asymptote of the curve $y = \frac{x^2 - 2x + 1}{2x + 3}$. **2**
8. If $f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2 + y^2}}, & (x, y) \neq (0,0) \\ 0, & (x, y) = (0,0) \end{cases}$, find $\frac{\partial f}{\partial x}$ at (0, 0). **3**
9. Evaluate $\iint_R e^{x^2} dx\, dy$, $R : 2y \le x \le 2, 0 \le y \le 1$. **3**
10. Find the unit normal vector to the surface $x^3 + y^3 + 3xyz = 3$ at (1, 2, –1). **2**

PART – B (5×10=50 Marks)

11. a) Find the eigen values and the corresponding eigen vectors of

$$A = \begin{pmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{pmatrix}.$$ 5

b) State and prove Cayley – Hamilton theorem. 5

12. a) Test for convergence of the series

$$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + ...$$ 5

b) Examine the series $\sum \frac{\cos n\pi}{x^2+1}$ for convergence. 5

13. a) State and prove Cauchy's mean value theorem. 5

b) Find the radius of curvature of $r = a \sin z\theta$ at $\theta = \pi/4$. 5

14. a) Find the total differential of the function $z = \tan^{-1}(x/y), (x, y) \neq (0,0)$. 5

b) Find the linear Taylor series polynomial approximation to the function $f(x, y) = 2x^3 + 3y^3 - 4x^2y$ about (1, 2). 5

15. Verify Green's theorem for $M(x, y) = e^{-x} \sin y$, $N(x, y) = e^{-x} \cos y$ and C is the square with vertices at (0, 0), $\left(\pi/2, 0\right)$, $\left(\pi/2, \pi/2\right)$, $\left(0, \pi/2\right)$. 10

16. a) Reduce the quadratic form Q = 2 (xy + yz + zx) in to canonical form. 5

b) Discuss the nature of the series

$$\frac{1}{2} + \frac{2}{3}x + \left(\frac{3}{4}\right)^2 x^2 + \left(\frac{4}{5}\right)^3 x^3 + ..., x > 0.$$ 5

17. a) If $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$. 5

b) Show that $\nabla^2(r^n) = n(n+1)r^{n-2}$. 5