Code No. 14522/CBCS

FACULTY OF ENGINEERING

B.E. (Except I.T.) III-Semester (CBCS) (Backlog) Examination, July 2021

Subject : Engineering Mathematics - III

Max. Marks: 70

Time: **2 hours** **Note: Missing data, if any, may be suitably assumed.**

PART – A

(5x2 = 10 Marks)

Answer any five questions.

1 Determine 'p' such that $f(z) = \frac{1}{2}\log(x^2 + y^2) + i\tan^{-1}\left(\frac{px}{y}\right)$ is an analytic function.

2 Evaluate $\oint_C \frac{e^{2z}}{(z+2)^4}\,dz$ where C is the circle | z | = 3.

3 Find the residue of $f(z) = \frac{z^3}{(z-1)^4(z-2)(z-3)}$ at its pole z = 2.

4 Classify singularity of the function $f(z) = \frac{e^{1/z}}{z^2}$.

5 State Dirichlet's conditions for the existence of Fourier series.

6 Find b_n in the Fourier series expansion of $f(x) = 2x - x^2$ in (0, 3).

7 Find the partial differential equation by eliminating the arbitrary function f from $z = f(x^2 - y^2)$.

8 Solve $xp + yq = 3z$.

9 Solve $z = px + qy + \sqrt{1 + p^2 + q^2}$.

10 Find a particular integral of the equation

$$\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^2 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial y^3} = e^{x+2y}$$

PART – B

Answer any four questions. **(4x15 = 60 Marks)**

11 (a) Find the analytic function $f(z) = u + iv$ where $v(x, y) = e^{-x}(x\cos y + y\sin y)$.

(b) If $F(a) = \oint_C \frac{4z^2 + z + 5}{z - a}dz$ where C is the ellipse $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$, then find,

(i) F(3.5) (ii) F(i) (iii) F'(-1) (iv) F''(- i)

12 (a) Find the Laurent's series expansion of $f(z) = \frac{z^2 - 6z - 1}{(z-1)(z-3)(z+2)}$ in the region $3 < |z+2| < 5$.

(b) Evaluate $\int_{-\infty}^{\infty} \frac{dx}{x^4 + 1}$.

13 Obtain half range Fourier cosine series of the function

$$f(x) = \begin{cases} kx, & 0 \le x \le l/2 \\ k(l-x), & l/2 \le x \le l \end{cases}$$

Hence find the sum $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$

14 (a) Solve $x(y-z)p + y(z-x)q = z(x-y)$.

(b) Solve $2xz - px^2 - 2qxy + pq = 0$ by Charpit's method.

15 Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with boundary conditions $u(x, 0) = 3\sin\pi x$, and $u(0, t) = u(1, t) = 0$ where $0 < x < 1$, $t > 0$.

16 (a) Show that the bilinear transformation $w = \frac{2z+3}{z-4}$ maps the circle $x^2 + y^2 - 4x = 0$ into the line $4u + 3 = 0$.

(b) Using Cauchy's residue theorem evaluate $\oint_C \frac{z}{(z-1)(z-2)^2}$ where C is the circle $|z-2| = \frac{1}{2}$

17 (a) Solve

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin x.$$

(b) Solve $z^2 = pq$.

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