Code No: E-5601/N/AICTE

FACULTY OF ENGINEERING

B.E. I Semester (AICTE) (Main & Backlog) (New) Examination, February/ March 2023

Subject: Mathematics- I

Time: 3 Hours **Max. Marks: 70**

Note: (i) First question is compulsory and answer any four questions from the remaining six questions. Each questions carries 14 Marks.
(ii) Answer to each question must be written at one place only and in the same order as they occur in the question paper.
(iii) Missing data, if any, may be suitably assumed.

1. a) Define a sequence with an example.
   b) Find the envelope of a family of curves $y = cx + \frac{1}{c}$, where c is a parameter.
   c) If $x^3 + y^3 - 3xy = 0$, then find $\frac{d^2y}{dx^2}$.
   d) Evaluate $\int_0^1 \int_0^{x^2} e^{\frac{y}{x}} dydx.$ =
   e) Prove that $\nabla(logr) = \frac{\vec{r}}{r}$, where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $r = |\vec{r}|$.
   f) Discuss the applicability of Rolle's theorem for $f(x) = tanx$ in $[0, \pi]$.
   g) Show that $\vec{F} = yz\hat{\imath} + zx\hat{\jmath} + xy\hat{k}$ is solenoidal.

2. a) Examine the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{2^n + 3^n}$.
   b) Prove that the series $\sum(-1)^n \frac{cosnx}{n\sqrt{n}}$, $x \in \mathbb{R}$ is absolutely convergent.

3. a) State and prove Lagrange's mean value theorem.
   b) Find the centre of circle of curvature of the curve $xy = 1$ at $(1,1)$.

4. a) Discuss the continuity of the function $f(x,y) = \begin{cases} \frac{x^2-y^2}{x^2+4y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ at the point (0,0).
   b) Using Lagrange's method of multipliers, find the maximum distance of the point (3,4,12) from the sphere $x^2 + y^2 + z^2 = 1$.

5. a) Evaluate $\iint_R \frac{x^2y^2}{x^2+y^2} dxdy$ over the annular region enclosed between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ by changing to polar coordinates.
   b) Evaluate $\int_0^1 \int_0^x \int_0^{x+y} e^{x+y+z} dz\, dy dx.$

6. Verify Green's theorem in the plane for $\int_C (\overset{M}{2xy - x^2})dx + (\overset{N}{x^2 + y^2})dy$, where C encloses the region R bounded by $y = x^2$ and $y^2 = x$.

7. (a) Discuss the convergence of the series $1 + \overset{1}{\frac{x}{2}} + \frac{x^2}{5} + \frac{x^3}{10} + \cdots (x > 0)$.

(b) Find the directional derivative of $f(x, y, z) = xy + yz + zx$ in the direction of the vector $-2\hat{i} + \hat{j} + 2\hat{k}$ at $(1,2,0)$. 1

**