OSMANIA UNIVERSITY
FACULTY OF ENGINEERING
UNIVERSITY COLLEGE OF ENGINEERING (AUTONOMOUS)
B.E. **(All Branches) I-Semester** (Main) Examinations March/April 2022

MATHEMATICS - I

Time : 3 hours **Max. Marks: 70**

Note : *i) Each question carries **14** Marks.*
*ii) **First Question** is compulsory and answer all sub questions. Answer any **four questions** from the remaining **six** questions **(Q2-7Q)**.*
iii) Answers to each question must be written at one place only and in the same order as they occur in the question paper.
iv) Missing data, if any, may suitably be assumed.

Q	Question	Marks	BT	CO
1 a)	Define Integral Test.	2	2	1
b)	Find the envelope of the family of straight lines $y = mx + \sqrt{a^2m^2 + b^2}$ where "m" is a Parameter.	2	5	2
c)	If $u = \frac{x+y}{1-xy}$ *and* $v = \tan^{-1} x + \tan^{-1} y$, *find* $\frac{\partial(u,v)}{\partial(x,y)}$. Are u and v functionally related? If so, find this relationship.	2	4	3
d)	Using spherical polar coordinates, find the volume of the sphere $x^2 + y^2 + z^2 = a^2$	2	5	4
e)	Find $curl\ (grad\ \phi)$	2	4	5
f)	Discuss the applicability of Cauchy's mean value theorem for the functions $f(x)=\frac{1}{x^2},\ g(x) = \frac{1}{x}$ *on* $[a ,b]$	2	3	2
g)	Evaluate the triple integral $\int_0^1\int_y^1\int_0^{1-x} x\, dz\, dx\, dy$	2	2	4
2.a)	Find the half-range cosine series of $f(x)=x,\ 0 \prec x \prec \pi$ and hence show that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+ \ldots\ldots\ldots\ldots = \frac{\pi^2}{8}$	8	6	1
b)	Test the convergence of the series : $1+\frac{1}{2}x+\frac{1.3}{2.4}x^2+\frac{1.3.5}{2.4.6}x^3 + \ldots\ldots\ldots\ldots$	6	4	1
3.a)	Prove that $\frac{\pi}{3}-\frac{1}{5\sqrt{3}} \succ \cos^{-1}\frac{3}{5} \succ \frac{\pi}{3} - \frac{1}{8}$ using Lagrange's mean value theorem	7	5	2
b)	Find the radius of curvature at any point on the curve $r^n = a^n \cos n\theta$	7	3	2

4.a)	Use the method of the Lagrange's multipliers to find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	8	2	3
b)	If $x = r\sin\theta\cos\phi, y = r\sin\theta\sin\phi, z = r\cos\theta$ show that $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = r^2\sin\theta$	6	4	3
5.a)	Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	8	3	4
b)	Show that the area between the parabolas $y^2 = 4ax$ *and* $x^2 = 4ay$ *is* $\frac{16}{3}a^2$	6	2	4
6.a)	Evaluate $\int_s \overline{F}\cdot\overline{n}\,ds$ where $\overline{F} = 18z\overline{i} - 12\overline{j} + 3y\overline{k}$ and S is the part of the surface of the plane $2x+3y+6z=12$ located in the first octant.	7	4	5
b)	Prove that $\nabla^2(r^m) = m(m+1)r^{m-2}$	7	3	5
7.a)	Find the maximum value of $u = x^2y^3z^4$ if $2x+3y+4z=a$	6	6	3
b)	Use Gauss divergence theorem to evaluate. $\iint_S\left(yz^2\,\overline{i}+zx^2\,\overline{j}+2z^2\,\overline{k}\right)\overline{ds}$, where S is the closed surface bounded by the xy- plane and the upper half of the sphere $x^2+y^2+z^2=a^2$ above this plane.	8	4	5

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