Code No. 1521 / E

FACULTIES OF ARTS AND SCIENCE

B.A./ B.Sc. III-Year Examination, March / April 2014

Subject : Mathematics
Paper – IV(b) : Fourier Series and Integral Transforms

Time : 3 Hours **Max. Marks : 100**

Note: Answer six questions from Section-A and four questions from Section-B, choosing atleast one from each unit. Each question in Section-A carries six marks and in Section-B carries 16 marks.

Section – A (6 x 6 = 36 Marks)

Unit-I

1 Find the Fourier series expansion for $f(x)=\begin{cases}-\pi, & -\pi<x<0\\ x, & 0<x<\pi\end{cases}$

2 Find the Fourier series expansion of f(x) = |x| in ($-\pi$, π).

Unit-II

3 Find L{t(3sin2t - 2 cos 2t)}.

4 Find $L^{-1}\left\{\dfrac{p-1}{(p+3)(p^2+2p+2)}\right\}$.

Unit-III

5 Find the cosine transform of $f(x)=\begin{cases}\cos x, & 0<x<a\\ 0, & x>a\end{cases}$

6 Find the Fourier cosine transform of e^{-x^2}.

Unit-IV

7 Solve (D^2 + 4D + 4)x = sinwt, t > 0 with x_o and x_1 for values of x and Dx when t = 0

8 Solve t y'' + y' + 4ty = 0 if y(0)=3, y'(0)=0.

Section-B (4 x 16 = 64 Marks)

Unit-I

9 (a) Find the Fourier series expansion for f(x) = xsinx in the interval ($-\pi$, π). Hence deduce that $\pi/4=\frac{1}{2}+\frac{1}{1.3}-\frac{1}{1.5}+....$

(b) Expression f(x) = x as a half-range sine and cosine series in 0 < x < 2.

10 (a) Find the Fourier series for $f(x)=\begin{cases}0, & -2<x<0\\ 1, & 0<x<2\end{cases}$

(b) Expand f(x) = e^{-x} as a Fourier series in the interval ($-\ell$, ℓ).

Unit-II

11 (a) If L^{-1}\{f(p)\} = F(t) then show that $L^{-1}\{f^n(p)\}=L^{-1}\left[\dfrac{d^n f(p)}{dp^n}\right]=(-1)^n t^n F(t)$. Using it find $L^{-1}\left\{\log\left(\dfrac{p+3}{p+3}\right)\right\}$.

(b) State and prove Initial value theorem and Final value theorem.

12 (a) If L{F(t)} = f(p), then show that $L\left\{\frac{1}{t}F(t)\right\}=\int_p^\infty f(x)dx$, and hence find $L\left\{\dfrac{\sin t}{t}\right\}$.

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(b) State and prove Heaviside's expansion theorem, and use it to find $L^{-1}\left[\dfrac{2p^2+5p-4}{p^3+p^2-2p}\right]$.

Unit-III

13 (a) Find the Fourier transform of $F(x)=\begin{cases}1-x^2, & |x|\le 1\\ 0. & |x|>1\end{cases}$ and hence evaluate $\int_0^\infty\left(\dfrac{x\cos x-\sin x}{x^3}\right)\cos\dfrac{x}{2}dx$.

(b) Find the Fourier sine transform of $\dfrac{e^{-ax}}{x}$

14 (a) Find the finite Fourier sine and cosine transform of f(x) = x.

(b) Find the Fourier sine transform of $f(x)=\dfrac{1}{x(a^2+x^2)}$.

Unit-IV

15 (a) Solve $(D^3 - D^2 + 4D - 4)y = 68\,e^t \sin 2t$, y = 1, Dy = -19, D^2y = -37 at t = 0.

(b) Solve $\dfrac{\partial u}{\partial t}=\dfrac{\partial^2 u}{\partial x^2}$ $\;if\; u_x(0,t)=0, u(x,0)=\begin{cases}x, & 0\le x\le 1\\ 0, & x>1\end{cases}$ and u(x, t) is bounded when x > 0, t > 0, use Fourier cosine transform.

16 (a) Solve $(D^2 - 2)x - Dy = 1$, $Dx + (D^2 + 2)y = 0$ if x = 0 = Dx = Dy = y when t = 0.

(b) Solve $\dfrac{\partial u}{\partial t}=2\dfrac{\partial^2 u}{\partial x^2}$ if u(0, t), u(x, 0) = e^{-x}, x > 0, u(x, t) is bounded when x > 0, t > 0.

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