

**FACULTY OF ENGINEERING**  
**B.E. 2/4 (CSE) First Semester (Suppl.) Examination, June/July 2011**  
**DISCRETE STRUCTURES**

Time : Three Hours]

[Maximum Marks : 75

**Note :—** Answer **ALL** questions from Part A. Answer any **FIVE** questions from Part B.

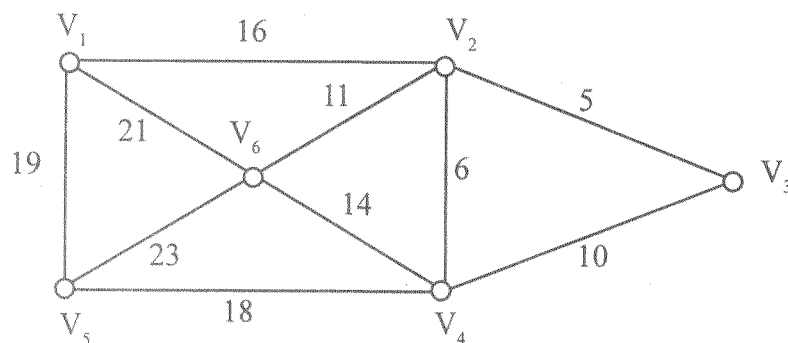
**PART—A (Marks : 25)**

1. Define Converse, Contrapositive and Inverse of an Implication. 3
2. Define Quantifiers. 2
3. What is partial order relation ? 2
4. What is pigeonhole principle ? 2
5. Define semigroups and monoids. 3
6. What is inhomogeneous recurrence relation ? 2
7. What is complete Bipartite graph ? 3
8. What is algebraic system ? 2
9. What is composition of functions ? 3
10. Find chromatic number of a wheel graph. 3

**PART—B (Marks : 50)**

11. (a) Show that  $P \rightarrow (Q \rightarrow P) \Leftrightarrow \neg P \rightarrow (P \rightarrow Q)$ . 4
- (b) Show that  $Q \vee (P \wedge \neg Q) \wedge (\neg P \wedge \neg Q)$  is a tautology. 6
12. (a) On the set  $Z$ , define the relation  $R$  by  $aRb$  if and only if  $ab \geq 0$ . Prove that  $R$  is reflexive and symmetric, but not transitive. 5
- (b) Consider the functions  $f$  and  $g$  defined by  $f(x) = x^3$ ,  $g(x) = x^2 + 1$ ,  $\forall x \in R$ . Find  $g \circ f$ ,  $f \circ g$ ,  $f^2$  and  $g^2$ . 5

13. (a) State and explain the properties of the pigeonhole principle. 5
- (b) Find the number of derangements of the integers from 1 to 10 inclusive, satisfying the condition that the set of elements in a first-5 places, is :
- (i) 1, 2, 3, 4, 5 in some order
- (ii) 6, 7, 8, 9, 10 in some order. 5
14. (a) Solve the recurrence relation  $a_r = 3a_{r-1} + 2$ ,  $r \geq 1$ ,  $a_0 = 1$  using generating function. 5
- (b) Solve the recurrence relation  $a_n - 9a_{n-1} + 26a_{n-2} - 24a_{n-3} = 0$  for  $n \geq 3$  with  $a_0 = 0$ ,  $a_1 = 1$ ,  $a_2 = 10$ . 5
15. (a) If  $G$  is a group such that  $(ab)^m = a^m b^m$  for three consecutive integers  $m$  for all  $a, b \in G$ , show that  $G$  is abelian. 5
- (b) Let  $G$  be a group and  $H$  a subgroup of  $G$ . Let  $f$  be an automorphism of  $G$  and  $f(H) = \{f(h) \mid h \in H\}$ . Prove that  $f(H)$  is a subgroup of  $G$ . 5
16. (a) Use Kruskal's algorithm to find minimum spanning tree : 6



- (b) Define the following graphs with an example each :
- (i) Complement graph
- (ii) Subgraph
- (iii) Spanning subgraph. 4
17. (a) Prove that a complete graph  $K_n$  is planar iff  $n \leq 4$ . 5
- (b) Write a brief note about the basic rules for constructing Hamiltonian cycles. 5